Matricial Algebra and Numerical Analysis MANA

CM 1 - Review on linear alebra and isometries

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01

Review on linear algebra

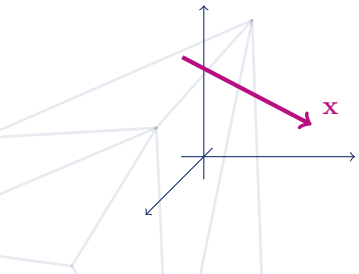
What is a vector?

Definition

Vectors are particular object for which the addidtion or the multiplication with a scalar generate an object of the same type.

Examples:

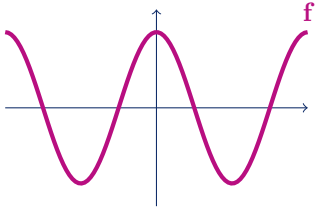
A vector $\mathbf{x} \in \mathbb{R}^3$.



An image $\mathbf{y} \in [0, 255]^{M \times N \times 3}$



A fonction $\mathbf{f} \in \mathcal{C}^2(\mathbb{R})$



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Definition

A vector space of $E = \mathbb{R}$ or $\mathbb{C}$, is a set V with an additive internal composition law $+ : V \times V \to V$ and a multiplicative external composition law $. : E \times V \to V$ verifying:

- Internal law property: $\forall (\mathbf{x}, \mathbf{y}, \mathbf{z}) \in V^3$,
  - commutativity: $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$
  - associativity:$\mathbf{x} + (\mathbf{y} + \mathbf{z}) = (\mathbf{x} + \mathbf{y}) + \mathbf{z}$
  - it exists a neutral element $0_V \in V$ such that $\mathbf{x} + 0_V = \mathbf{x}$.
- External law property: $\forall (\mathbf{x}, \mathbf{y}) \in V^2, \forall (\lambda, \mu) \in E^2$,
  - linearity: $\lambda(\mu \mathbf{x}) = (\lambda\mu)\mathbf{x}$,
  - left distributivity: $\lambda(\mathbf{x} + \mathbf{y}) = \lambda\mathbf{x} + \lambda\mathbf{y}$,
  - right distributivit: $(\lambda + \mu)\mathbf{x} = \lambda\mathbf{x} + \mu\mathbf{x}$,
  - it exists a neutral element $1_E \in E$ such that $1_E \mathbf{x} = \mathbf{x}$, $\forall \mathbf{x} \in V$.
- Stability: $\forall (\mathbf{x}, \mathbf{y}) \in V^2, \forall (\lambda, \mu) \in E^2$, $(\lambda\mathbf{x} + \mu\mathbf{y}) \in V$.

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Linear independence

A set of vectors $\{\mathbf{e}_i\}_{i=1:n}$ of V is linearly independent if

$$\sum_{i=1}^{n} \lambda_i \mathbf{e}_i = 0_V \Rightarrow \lambda_i = 0_E, \forall i = 1 : n. \tag{1}$$

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Spanning set of vectors

A set of vectors $\{\mathbf{e}_i\}_{i=1:n}$ of V is a spanning set of vectors if $\forall \mathbf{x} \in V$, $\exists(\lambda_1, \lambda_2, \dots, \lambda_n) \in E^N$ such that

$$\mathbf{x} = \sum_{i=1}^{n} \lambda_i \mathbf{e}_i. \tag{2}$$

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Basis

Linear independence

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Basis

A set of vectors $\{\mathbf{e}_i\}_{i=1:n}$ of V is a basis if it is a spanning set of linearly independent vectors.

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What is a matrix?

Definition

A matrix is a compact way to represent linear systems or linear functions. W

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Setting $(m, n) \in \mathbb{N}^2$, a matrix $\mathbf{A}$ and a $m \times n$-tuple of coefficients $(a_{i,j})_{i=1:m,j=1:n}$, members of a set E, ordered in a rectangulare scheme of m rows and n colomns :

$$\begin{pmatrix} a_{1,1} & \dots & a_{1,j} & \dots & a_{1,n} \\ \vdots & \ddots & & & \vdots \\ a_{i,1} & & a_{i,j} & & a_{i,n} \\ \vdots & & & \ddots & \vdots \\ a_{m,1} & \dots & a_{m,j} & \dots & a_{m,n} \end{pmatrix}. \tag{3}$$

We note $\mathbf{A} \in \mathcal{M}_{m,n}(E)$ (French notation) or $\mathbf{A} \in E^m \times E^n$ or $\mathbf{A} \in (E^m)^n$ (English notation).

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Matrix/Vector product

Let V_1, V_2 be two vector spaces. Setting $\mathbf{A} = (a_{i,j})_{i=1:m,j=1:n} \in \mathcal{M}_{m,n}(E)$ a matrix and $\mathbf{x} = (\mathbf{x}_j)_{j=1:n} \in V_1$ a vector, then the vector $\mathbf{y} \in V_2$ is such that:

$$\mathbf{y} = \mathbf{A}\mathbf{x} \Leftrightarrow \forall i = 1:m,\ \mathbf{y}_i = \sum_{j=1}^{n} a_{i,j}\mathbf{x}_j. \tag{4}$$

$$\begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_i \\ \vdots \\ \mathbf{y}_m \end{pmatrix} = \begin{pmatrix} a_{1,1} & \dots & a_{1,j} & \dots & a_{1,n} \\ \vdots & \ddots & & & \vdots \\ a_{i,1} & & a_{i,j} & & a_{i,n} \\ \vdots & & & \ddots & \vdots \\ a_{m,1} & \dots & a_{m,j} & \dots & a_{m,n} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_j \\ \vdots \\ \mathbf{x}_n \end{pmatrix}$$

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$$\begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_i \\ \vdots \\ \mathbf{y}_m \end{pmatrix} = \begin{pmatrix} a_{1,1} & \dots & a_{1,j} & \dots & a_{1,n} \\ \vdots & \ddots & & & \vdots \\ a_{i,1} & & a_{i,j} & & a_{i,n} \\ \vdots & & & \ddots & \vdots \\ a_{m,1} & \dots & a_{m,j} & \dots & a_{m,n} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_j \\ \vdots \\ \mathbf{x}_n \end{pmatrix}$$

WARNING!

If we want to apply the matrix two a row vector, then it must be applied on the right and the result is a row vector, but $\mathbf{x}^t\mathbf{A} \neq \mathbf{y}^t$! You should transpose $\mathbf{A}$ as well!

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Product between two matrices

Let $\mathbf{A} = (a_{i,j})_{i=1:\ell, j=1:m} \in \mathcal{M}_{\ell,m}(E)$ and $\mathbf{B} = (b_{j,k})_{j=1:m, k=1:n} \in \mathcal{M}_{m,n}(E)$ be two matrices, the the matrix $\mathbf{C} = (c_{i,k})_{i=1:\ell, k=1:n} \in \mathcal{M}_{\ell,n}(E)$ is such that:

$$\mathbf{C} = \mathbf{AB} \Leftrightarrow \forall i = 1:\ell, \forall k = 1:n,\ \mathbf{C}_{i,k} = \sum_{j=1}^{m} a_{i,j} b_{j,k}. \tag{5}$$

$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,j} & \cdots & a_{1,m} \\ \vdots & \ddots & & & \vdots \\ a_{i,1} & & a_{i,j} & & a_{i,m} \\ \vdots & & & \ddots & \vdots \\ a_{\ell,1} & \cdots & a_{\ell,j} & \cdots & a_{\ell,m} \end{pmatrix} \begin{pmatrix} b_{1,1} & \cdots & b_{1,k} & \cdots & b_{1,n} \\ \vdots & \ddots & & & \vdots \\ b_{j,1} & & b_{j,k} & & b_{j,n} \\ \vdots & & & \ddots & \vdots \\ b_{m,1} & \cdots & b_{m,k} & \cdots & b_{m,n} \end{pmatrix} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,k} & \cdots & c_{1,n} \\ \vdots & \ddots & & & \vdots \\ c_{i,1} & & c_{i,k} & & c_{i,n} \\ \vdots & & & \ddots & \vdots \\ c_{\ell,1} & \cdots & c_{\ell,k} & \cdots & c_{\ell,n} \end{pmatrix}$$

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$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,j} & \cdots & a_{1,m} \\ \vdots & \ddots & & & \vdots \\ a_{i,1} & & a_{i,j} & & a_{i,m} \\ \vdots & & & \ddots & \vdots \\ a_{\ell,1} & \cdots & a_{\ell,j} & \cdots & a_{\ell,m} \end{pmatrix} \begin{pmatrix} b_{1,1} & \cdots & b_{1,k} & \cdots & b_{1,n} \\ \vdots & \ddots & & & \vdots \\ b_{j,1} & & b_{j,k} & & b_{j,n} \\ \vdots & & & \ddots & \vdots \\ b_{m,1} & \cdots & b_{m,k} & \cdots & b_{m,n} \end{pmatrix} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,k} & \cdots & c_{1,n} \\ \vdots & \ddots & & & \vdots \\ c_{i,1} & & c_{i,k} & & c_{i,n} \\ \vdots & & & \ddots & \vdots \\ c_{\ell,1} & \cdots & c_{\ell,k} & \cdots & c_{\ell,n} \end{pmatrix}$$

WARNING !

$\mathbf{AB} \neq \mathbf{BA}$! The matricial product is not commutative!! Here, it is not even possible to proceed to the $\mathbf{BA}$ product!

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Some properties

- If $\mathbf{A}$ is invertible, its inverse is the matrix $\mathbf{A}^{-1}$ such that $\mathbf{A}\mathbf{A}^{-1} = \mathbf{Id}$.

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- The adjoint of the matrix $\mathbf{A} \in \mathcal{M}_{m,n}(E)$ defined earlier, is the matrix $\mathbf{A}^* \in \mathcal{M}_{n,m}(E)$, where * denote the adjoint, such that:

$$\mathbf{A}^* = \begin{pmatrix} \overline{a}_{1,1} & \dots & \overline{a}_{i,1} & \dots & \overline{a}_{m,1} \\ \vdots & \ddots & & & \vdots \\ \overline{a}_{1,j} & & \overline{a}_{i,j} & & \overline{a}_{m,j} \\ \vdots & & & \ddots & \vdots \\ \overline{a}_{1,n} & \dots & \overline{a}_{i,n} & \dots & \overline{a}_{m,n} \end{pmatrix}. \tag{6}$$

where $\overline{a}_{i,j}$ is the complex conjugate of $a_{i,j}$.

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- A matrix is called symmetric if $\mathbf{A} = \mathbf{A}^t$ or hermitian if $E \subset \mathbb{C}$ and $\mathbf{A} = \mathbf{A}^*$.
- A matrix is called orthogonal if $\mathbf{A}\mathbf{A}^t = \mathbf{Id}$ or unitary if $E \subset \mathbb{C}$ and $\mathbf{A}\mathbf{A}^* = \mathbf{Id}$.
- Finally, $(\mathbf{AB})^* = \mathbf{B}^*\mathbf{A}^*$.

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Sub-matrix et cofactor matrix

Setting the submatrix $\mathbf{A}_{i,j} = (a_{k,\ell})_{\substack{k=1:m,k\neq i \\ \ell=1:n,\ell\neq j}}$:

$$\mathbf{A}_{i,j} = \begin{pmatrix} a_{1,1} & \cdots & a_{1,j-1} & a_{1,j+1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{i-1,1} & \cdots & a_{i-1,j-1} & a_{i-1,j+1} & \cdots & a_{i-1,n} \\ a_{i+1,1} & \cdots & a_{i+1,j-1} & a_{i+1,j+1} & \cdots & a_{i+1,n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & \cdots & a_{m,j-1} & a_{m,j+1} & \cdots & a_{m,n} \end{pmatrix}. \tag{7}$$

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Cofactor matrix

The cofactore matrix $\mathbf{A}$ is define $\forall i = 1:m, \forall j = 1:n$ by :

$$\mathrm{com}(\mathbf{A})_{i,j} = (-1)^{i+j} \det(\mathbf{A}_{i,j}) \tag{8}$$

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Determinant

The determinant can be calculated recursively with the following formula :

$$\det(\mathbf{A}) = \sum_j (-1)^{1+j} a_{1,j} \det(\mathbf{A}_{1,j}) \tag{9}$$

Matricial inverse

The inverse of a matrix can be calculated using the cofactor matrix and using the following relation :

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \mathrm{com}(\mathbf{A})^t \tag{10}$$

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Tips and tricks for low dimensions

Even if yout computer can do the work for you, you should be able to do the calculs by hand for low dimension matrices. It is important if you have to debug some code.

2 Dimensions

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det(\mathbf{A}) = ad - cb$$

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

3 Dimensions

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\det(\mathbf{A}) =$$

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3 Dimensions

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\det(\mathbf{A}) = aei$$

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3 Dimensions

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\det(\mathbf{A}) = aei + bfg$$

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$$\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\det(\mathbf{A}) = aei + bfg + cdh$$

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02

Isometries

Isometries

Isometries are geometric transformations which conserve length and angles.





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Isometries are geometric transformations which conserve length and angles.



Rotation

$$\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Horizontal reflection

$$\mathbf{S}_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Vertical reflection

$$\mathbf{S}_y = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Translation

$$\mathbf{T}_\mathbf{v} = \begin{pmatrix} 1 & 0 & \mathbf{v}_1 \\ 0 & 1 & \mathbf{v}_2 \\ 0 & 0 & 1 \end{pmatrix}$$

Reflections

$$\mathbf{S}_\theta = \mathbf{R}_\theta \mathbf{S}_x \mathbf{R}_{-\theta} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

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Questions?

